AVIRAL
JEE | NEET | FOUNDATIONS

# WEEKLY TEST TYJ TEST - 22 B
# SOLUTION Date 22-09-2019

## [PHYSICS]

1. The co-ordinates of the corners of the square are (0, 0), (2, 0), (2, 2), (0, 2). Hence,

$$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{2\times 0 + 3\times 2 + 5\times 2 + 8\times 0}{2+3+5+8} = \frac{16}{18} = \frac{8}{9}\text{ m}$$

$$Y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{2\times 0 + 3\times 0 + 5\times 2 + 8\times 2}{2+3+5+8} = \frac{26}{18} = \frac{13}{9}\text{ m}$$

$\therefore$ Co-ordinates of the centre of mass $= \left(\frac{8}{9}, \frac{13}{9}\right)$.

2. Mass of the disc removed $= \frac{M}{\pi R^2} \times \pi\left(\frac{R}{2}\right)^2 = \frac{M}{4}$

Remaining mass $= M - \frac{M}{4} = \frac{3M}{4}$

Let the origin of the co-ordinate system coincide with the centre of mass of whole disc. Now, we know that;

$$X_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$X_{CM}$ will be zero, when

$$m_2x_2 = -m_1x_1$$

$\therefore$ $x_2 = -\frac{m_1}{m_2}x_1$

Here, $m_1 = \frac{M}{4}$, $x_1 = \frac{R}{2}$

and $m_2 = \frac{3M}{4}$ (for remaining mass)

Hence, $x_2 = -\frac{M/4}{3M/4}\cdot\frac{R}{2} = \frac{-R}{6}$

*i.e.*, $\frac{R}{6}$ from the centre (on LHS).

3. Let $\rho$ be the density of lead.

Then, $M = \frac{4}{3}\pi R^3 \rho$ = mass of total sphere

$$m_1 = \frac{4}{3}\pi \left(\frac{R}{2}\right)^3 \rho = \text{mass of removed part} = \frac{M}{8}$$

$$m_2 = M - \frac{M}{8} = \frac{7M}{8} = \text{mass of remaining sphere}$$

Choosing the centre of big sphere as the origin,

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$0 = \frac{\frac{M}{8} \times \frac{R}{2} + \frac{7M}{8} \times x_2}{M}$$

Solving, we get; $x_2 = \frac{-R}{14}$

*i.e.*, centre of mass of hollowed sphere would be at a distance of $R/14$ on left of $O$,

*i.e.*, shift in centre of mass $= \frac{R}{14}$.

4. Given that the system is initially at rest,

*i.e.*, $\vec{V}_{CM} = 0$

$\therefore$ $\frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} = 0$

or $m_1 \vec{v}_1 + m_2 \vec{v}_2 = 0$

or $m_1 \frac{\Delta \vec{r}_1}{\Delta t} + m_2 \frac{\Delta \vec{r}_2}{\Delta t} = 0$

or $m_1 \Delta \vec{r}_1 + m_2 \Delta \vec{r}_2 = 0$

Now, here in boat-man system if the man moves towards right the boat moves towards left.

$\therefore$ $m_1 \Delta r_1 = m_2 \Delta r_2$ ...(i)

($\because \Delta r_1$ is opposite to $\Delta r_2$)

If $\Delta r_2$ is the displacement of boat relative to shore, then the displacement of man relative to shore would be $(9 - \Delta r_2)$,

*i.e.*, $\Delta r_1 = 9 - \Delta r_2$ ...(ii)

From eqn. (i) and (ii),

$m_1 (9 - \Delta r_2) = m_2 \Delta r_2$

or $100(9 - \Delta r_2) = 500\, \Delta r_2$

$\therefore$ $\Delta r_2 = \frac{100 \times 9}{600} = 1.5$ m

*i.e.*, Boat moves 1.5 m relative to shore in the direction opposite to the displacement of the man.

5. Given that initially the system is at rest, so

$$\vec{V}_{CM} = 0$$

Now, as in the motion of dog no external force is applied to the system, hence

$$\vec{V}_{CM} = \text{constant} = 0$$

i.e., $\frac{m\vec{v}_1 + M\vec{v}_2}{m+M} = 0$

or $m\vec{v}_1 + M\vec{v}_2 = 0$ [as $(m+M)$ = finite]

or $m\frac{\Delta\vec{r}_1}{\Delta t} + M\frac{\Delta\vec{r}_2}{\Delta t} = 0$

or $m\,\Delta\vec{r}_1 + M\,\Delta\vec{r}_2 = 0$ (as $\Delta\vec{r} = \vec{d}$ = displacement)

or $md_1 - Md_2 = 0$ (as $\vec{d}_2$ is opposite to $\vec{d}_1$)

or $md_1 = Md_2$ ...(i)

Now, when dog moves 4 m towards shore relative to boat, the boat will shift a distance $d_2$ relative to shore opposite to the displacement of dog; so, the displacement of dog relative to shore (towards shore) will be,

$d_1 = 4 - d_2$ ...(ii)

(i.e., $d_1 + d_2 = d_{rel.} = 4$).

Putting the value of $d_2$ from eqn. (ii) in eqn. (i),

$md_1 = M(4 - d_1)$

or $d_1 = \frac{M \times 4}{m + M} = \frac{20 \times 4}{5 + 20} = 3.2$ m

As initially the dog was 10 m from the shore, so now he will be $(10 - 3.2) = 6.8$ m from the shore.

6. Refer to question 13 Two particles collide at their centre of mass.

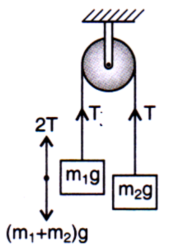
$\therefore$ Distance of CM from $P$

$$= \frac{0.1 \times 0 + 0.3 \times 1}{0.1 + 0.3} = 0.75 \text{ m}.$$

7. Under mutual attraction, the centre of mass remains at rest *i.e.*, velocity is zero.

8. The equation of motion of the centre of mass is,

$$M\vec{a}_{CM} = \vec{F}_{ext.}$$




And as there is no external force in horizontal direction, so the centre of mass of the system does not change along horizontal direction.

For vertical motion of the centre of mass,

$$(a_{CM})_y = \frac{F_{ext.}}{M} = \frac{(m_1 + m_2)g - 2T}{(m_1 + m_2)} \quad ...(i)$$

Further, $a_{CM} = \frac{m_1a_1 + m_2a_2}{m_1 + m_2}$

$= \frac{m_1 - m_2}{m_1 + m_2}a$ [$\because \vec{a}_1 = a$ and $\vec{a}_2 = -a$]

$= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$ $\left[\because a = \frac{m_1 - m_2}{m_1 + m_2}g\right]$

However, the equations of motion of two blocks are

$m_2g - T = m_2a$

$T - m_1g = m_1a$

Eliminating $a$, we get;

$$T = \frac{2m_1m_2}{m_1 + m_2}g \quad ...(ii)$$

Putting eqn. (ii) in eqn. (i), we get;

$$(a_{CM})_y = \frac{(m_1 + m_2)^2 - 4m_1m_2}{(m_1 + m_2)^2}g = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$$

[**Note:** It is clear that acceleration of CM is always vertically downward irrespective of whether $m_1$ is heavier or $m_2$.]

9. Force $F_A$ on the particle $A$ is given by:

$$F_A = m_A\, a_A = \frac{m_A v}{t} \qquad \ldots(i)$$

Similarly, $$F_B = m_B\, a_B = \frac{m_B \times 2v}{t} \qquad \ldots(ii)$$

Now, $$\frac{m_A v}{t} = \frac{m_B \times 2v}{t} \qquad (\because F_A = F_B)$$

So, $$m_A = 2m_B$$

Hence, the speed of the centre of mass of the system

$$V_{CM} = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{2m_B\, v - m_B \times 2v}{2m_B + m_B} = 0 .$$

10. Here, the centre of mass of the system remains unchanged when the mass $m$ moved a distance $L\cos\theta$, let the mass $(m + M)$ moves a distance $x$ in the backward direction.

$\therefore$ $$(M + m)x - mL\cos\theta = 0$$

$\therefore$ $$x = (mL\cos\theta)/(m + M) .$$

11. $$R_{CM} = \frac{12 \times 0 + 16 \times 1.12 \times 10^{-10}}{12 + 16}$$

$$= \frac{16}{28} \times 1.12 \times 10^{-10}\ \text{m} = 0.64 \times 10^{-10}\ \text{m} .$$

12. $m_1 = 10$ kg, $m_2 = 2$ kg

$$\vec{v_1} = 2\hat{i} - 7\hat{j} + 3\hat{k}$$

$$\vec{v_2} = -10\hat{i} + 35\hat{j} - 3\hat{k}$$

$$\vec{v}_{CM} = \frac{m_1\vec{v_1} + m_2\vec{v_2}}{m_1 + m_2}$$

$$= \frac{10(2\hat{i} - 7\hat{j} + 3\hat{k}) + 2(-10\hat{i} + 35\hat{j} - 3\hat{k})}{10 + 2} = 2\hat{k}\ \text{m/s}$$

13. $$a_{CM} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$$

$$m_1 = m_2 = m$$

$$a_1 = 0; \quad a_2 = a$$

$\therefore$ $$a_{CM} = \frac{ma}{2m} = \frac{a}{2} .$$

14.

15. $$a = \frac{3m - m}{3m + m} g = \frac{g}{2}$$

Acceleration of centre of mass

$$= \frac{3m \times \frac{g}{2} - \frac{mg}{2}}{3m + m} = \frac{g}{4} .$$